Pearson Edexcel

Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for this paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- * – The answer is printed on the paper
- ⎿ – The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao), unless shown, for example as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Pure Mathematics Marking
(But note that specific mark schemes may sometimes override these general priniciples)

Method mark for solving 3 term quadratic:

1. **Factorisation**
   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = \ldots$
   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = \ldots$
2. **Formula**
   Attempt to use the correct formula (with values for *a*, *b* and *c*).
3. **Completing the square**
   Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, q \neq 0$, leading to $x = \ldots$

Method mark for differentiation and integration:

1. **Differentiation**
   Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. **Integration**
   Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values. Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1 (a)	$\frac{5x+10}{(1-x)(2+3x)} \equiv \frac{A}{1-x}+\frac{B}{2+3x} \Rightarrow$ Value for A or B	**M1**
	One correct value, either $A=3$ or $B=4$	**A1**
	Correct PF form $\frac{3}{1-x}+\frac{4}{2+3x}$	**A1**
		(3)
(b)(i)	$\frac{A}{1-x}=A(1-x)^{-1}=A\left(1+x+x^2+\ldots\right)$	**B1**
	$\left\{\frac{B}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1}=\left\{\frac{B}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\ldots\right);=\frac{B}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)$	**M1; A1**
	$\text{f}(x)=\quad 3\times\left(1+x+x^2+\ldots\right)+\quad\frac{4}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)$	**M1**
	$=5+\frac{15}{2}x^2+\ldots$	**A1**
		(5)
(b)(ii)	$\lvert x\rvert<\frac{2}{3}$	**B1**
		(1)
		(9 marks)
(b)(i) Alt 1	$(1-x)^{-1}=1+x+x^2+\ldots$	**B1**
	$\left\{\frac{1}{2}\right\}\left(1+\frac{3x}{2}\right)^{-1}=\left\{\frac{1}{2}\right\}\left(1+(-1)\frac{3x}{2}+\frac{(-1)(-2)}{2}\left(\frac{3x}{2}\right)^2+\ldots\right);=\frac{1}{2}\left(1-\frac{3x}{2}+\frac{9}{4}x^2+\ldots\right)$	**M1; A1**
	$\frac{5x+10}{(1-x)(2+3x)}=(5x+10)\left(1+x+x^2+\ldots\right)\times\frac{1}{2}\left(1-\frac{3x}{2}+\frac{9x^2}{4}+\ldots\right)=5+\ldots x+\ldots x^2$	**M1**
	$=5+\frac{15}{2}x^2+\ldots$	**A1**
		(5)
(b)(i) Alt 2	$\frac{5x+10}{(1-x)(2+3x)}=(5x+10)\left(2+\left(x-3x^2\right)\right)^{-1}=\frac{1}{2}(5x+10)\left(1+\frac{1}{2}\left(x-3x^2\right)\right)^{-1}$	**B1**
	$(1+p(x))^{-1}=\left(1\pm p(x)+\frac{(-1)(-2)}{2}(p(x))^2+\ldots\right);\ \frac{1}{2}\left(1-\frac{1}{2}\left(x-3x^2\right)+\frac{1}{4}\left(x-3x^2\right)^2+\ldots\right)$	**M1; A1**
	$(10+5x)\left(\frac{1}{2}-\frac{1}{4}x+\frac{3}{4}x^2+\frac{1}{8}x^2+\ldots\right)=5-\frac{5}{2}x+\frac{35}{4}x^2+\frac{5}{2}x-\frac{5}{4}x^2+\ldots$	**M1**

	$=5+\frac{15}{2}x^2+\ldots$	**A1**
		(5)

Notes:

a)
M1: Attempts at correct PF. Correct form identified (may be implicit) and achieves a value for at least one of the constants.
A1: One correct value or term.
A1: Correct PF form $\frac{3}{1-x}+\frac{4}{2+3x}$. This may be awarded if seen in (b) but the correct final form (not just values) must be seen somewhere in the question. Accept at $3(1-x)^{-1}+4(2+3x)^{-1}$

(b)(i)
B1: $\frac{A}{1-x}=A(1-x)^{-1}=A\left(1+x+x^2+\ldots\right)$ which may be unsimplified. Allow with their A or with $A=1$.
M1: Attempts to expand $\frac{1}{2+3x}=(2+3x)^{-1}$ binomially either by taking out the factor 2 first, or directly. Look for $(1+kx)^{-1}=\ldots\left(1\pm kx+\frac{(-1)(-2)}{2}(kx)^2+\ldots\right)$ where $k\neq1$ following an attempt at taking out a factor 2, or $\frac{1}{2+3x}=(2+3x)^{-1}=\left(2^{-1}\pm2^{-2}kx+\frac{(-1)(-2)}{2}2^{-3}(kx)^2+\right)$ by direct expansion. Allow missing brackets on kx^2 in either case.
A1: $\frac{B}{2+3x}=\frac{B}{2}\left(1+\frac{3x}{2}\right)^{-1}=\frac{B}{2}\left(1-\frac{3x}{2}+\frac{9}{4}x^2+\right)$ oe with their B from (a) or with $B=1$
M1: Uses their coefficients and attempts to add both series.
A1cao: $5+\frac{15}{2}x^2+\ldots$ Condone additional higher order terms. Terms may be either order.

(b)(ii)
B1: $|x|<\frac{2}{3}$ or exact equivalent. This must be clearly identified as the answer. B0 if both ranges are given with no choice of which is correct. (But B1 if formal set notation with $\cap$ used.)

(b)(i) Alt 1:
B1: $(1-x)^{-1}=1+x+x^2+\ldots$ which may be unsimplified.
M1: Same as main scheme.
A1: Correct expansion (see main scheme, $B=1$ allowed).
M1: Attempts to expand all three brackets, achieving the correct constant term at least.
A1cso: $5+\frac{15}{2}x^2+\ldots$ Condone additional higher order terms. Terms may be either order.

(b)(i) Alt 2
B1: Writes f(x) as $(5x+10)\left(2+\left(x-3x^2\right)\right)^{-1}$ or with the 2 extracted, with the $\left(x-3x^2\right)$ clear.

M1: Attempts the binomial expansion on $\left(1+p(x)\right)^{-1}$ or $\left(2+p(x)\right)^{-1}$ for $p(x)$ of form $ax+bx^2$. Same conditions as for main scheme.

A1: Correct expansion. For direct expansion $\left(\frac{1}{2}-\frac{1}{4}\left(x-3x^2\right)+\frac{1}{8}\left(x-3x^2\right)^2+\ldots\right)$

M1: Expands the brackets achieving at least the correct constant term.

A1cao: $5+\frac{15}{2}x^2+\ldots$ Condone additional higher order terms. Terms may be either order.

Question Number	Scheme	Marks
2 (a)	E.g. $x=\dfrac{t-1}{2t+1}\Rightarrow t=\dfrac{x+1}{1-2x}$ or $y=\dfrac{6}{2t+1}\Rightarrow t=\dfrac{6-y}{2y}$	**M1**
	E.g. $y=\dfrac{6}{2t+1}\Rightarrow y=\dfrac{6}{2\times\left(\dfrac{x+1}{1-2x}\right)+1}$ or $t=\dfrac{6-y}{2y}\Rightarrow x=\dfrac{\dfrac{6-y}{2y}-1}{2\times\dfrac{6-y}{2y}+1}$	**A1**
	E.g. $y=\dfrac{6}{2\times\left(\dfrac{x+1}{1-2x}\right)+1}\Rightarrow y=\dfrac{6(1-2x)}{2\times(x+1)+1(1-2x)}=ax+b$	**dM1**
	E.g. $y=\dfrac{6(1-2x)}{3}, y=2(1-2x)$ oe so linear *	**A1***
		(4)
(b)	$y=2(1-2x)$ and $y=x+12\Rightarrow\ 2(1-2x)=x+12\Rightarrow x=...$	**M1**
	$x=-2$	**A1cao**
		(2)
Alt (b)	$\dfrac{6}{2t+1}=\dfrac{t-1}{2t+1}+12\Rightarrow t=\left(-\dfrac{1}{5}\right)$	**M1**
	$x=\dfrac{-\frac{1}{5}-1}{2\times-\frac{1}{5}+1}=-2$	**A1**
		(2)
		(6 marks)

Notes:

(a) Do not recover marks for part (a) from part (b) if there is an attempt at part (a). If there is no labelling mark as a whole.

M1: For an attempt to get *t* in terms of *x* or *y* **or** *x* and *y* $\dfrac{x}{y}=\dfrac{t-1}{6}\Rightarrow t=\dfrac{6x+y}{y}$ or full method to eliminate *t* from the equations.

A1: Forms a correct equation linking *x* and *y* only. Other forms are possible using *t* in terms of *x* and *y* in either equation for *x* or *y* etc.

dM1: Depends on first M. Attempts to simplify the fraction reaching a linear form in *x* and *y*. Allow if there are slips but an unsimplified equation of form $ax + by = c$ must be achieved.

A1: Achieves $y=2(1-2x)$ o.e. (and isw after a correct linear equation) and states linear or hence on line etc. There must be a reference to linearity in some form (similarly for the Alts).

(b)

M1: Solves their "$y=2(1-2x)$" (may not be linear) with $y=x+12$, E.g. $2(1-2x)=x+12\Rightarrow x=...$

A1: cao $x=-2$ (ignore any references to the y coordinate). Do not accept $-\frac{10}{5}$ Alt (b) M1: Solves the parametric equations simultaneously with the line equation to find a value for t A1: cao Deduces correct value for x.

2 (a) Alt 1	$\frac{dx}{dt}=\frac{(2t+1)-2(t-1)}{(2t+1)^2}$ and $\frac{dy}{dt}=\frac{-12}{(2t+1)^2}$ o.e.	**M1** **A1**
	$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	**dM1**
	$\frac{dy}{dx}=\frac{-12}{(2t+1)^2}\div\frac{3}{(2t+1)^2}=-4$ (which is a constant,) hence linear	**A1*** **(4)**
Alt 1 (a) via differentiation **Notes:**		
M1: Attempts $\frac{dx}{dt}$ and $\frac{dy}{dt}$ using appropriate rule for at least one. A1: Both correct dM1: Depends on first M. Attempts $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ leading to constant. A1: Achieves $\frac{dy}{dx}=-4$ and makes suitable conclusion e.g. "hence linear" *		
2 (a) Alt 2	$x=\frac{t-1}{2t+1}\Rightarrow x=A-\frac{B}{2t+1}$; $x=\frac{1}{2}-\frac{3}{2(2t+1)}$	**M1;** **A1**
	$x=\frac{1}{2}-\frac{3}{2\left(6/y\right)}$	**dM1**
	$x=\frac{1}{2}-\frac{1}{4}y$ hence linear *	**A1*** **(4)**
Alt 2 (a) via division **Notes:**		
M1: Attempts to write x in terms of just $2t+1$. E.g $x=\frac{t-1}{2t+1}\Rightarrow x=A-\frac{B}{2t+1}$. A1: $x=\frac{1}{2}-\frac{3}{2(2t+1)}$ dM1: Uses $y=\frac{6}{2t+1}$ to form an equation linking x and y A1: $x=\frac{1}{2}-\frac{1}{4}y$ and states linear*		
2 (a) Alt 3	$ax+by=\frac{at-a+6b}{2t+1}=\frac{k\left(2t+1\right)}{2t+1}\Rightarrow a=2k,6b-a=k$	**M1**
	$a=12b-2a\Rightarrow a=4b$	**A1**
	E.g. $4x+y=\frac{2\left(2t+1\right)}{2t+1}=...$	**dM1**
	$4x+y=2$ (oe) hence linear *	**A1*** **(4)**

Alt 3 (a) via elimination **Notes:**
M1: Writes $ax+by=...$ as a single fraction and attempts to compare coefficients of numerator with the denominator. A1: Correct ratio between a and b deduced. dM1: Uses their ratio to eliminate t from the equation. A1: $4x+y=2$ (oe) and states linear* Note: It is possible to spot the correct values for a and b directly so the following would gain full marks: $2x+\frac{1}{2}y=\frac{2t-2+3}{2t+1}=\frac{2t+1}{2t+1}=1$ hence linear. *

Question Number	Scheme	Marks
3.	States or implies Volume = $\int_{\sqrt{5}}^{5} \pi \left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$	**B1**
	$\int \left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x = \int \frac{3x}{(3x^2+5)} \mathrm{d}x = \frac{1}{2}\ln(3x^2+5)$	**M1A1**
	Volume = $\{\pi\}\left(\frac{1}{2}\ln(3\times 25+5) - \frac{1}{2}\ln(3\times 5+5)\right)$	**M1**
	$= \pi \ln 2$	**A1**
		(5 marks)

Notes:

B1: States or implies Volume = $\int_{\sqrt{5}}^{5} \pi \left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ o.e. The limits may be implied by subsequent work, and the dx may be missing. This is for knowing the correct formula rather than for notation. The π may be implied by later work.

M1: Attempts $\int \left(\sqrt{\frac{3x}{3x^2+5}}\right)^2 \mathrm{d}x$ to achieve $k\ln(3x^2+5)$ (oe). May use substitution, either u = $3x^2$ or $u = 3x^2+5$, in which case they should achieve $k\ln(u+5)$ or $k\ln(u)$ (oe) respectively. Allow if the brackets are missing.

A1: Correct result of integration, which may be left unsimplified. May be in terms of u if a substitution has been used. Allow if missing brackets are recovered, but A0 if never recovered.

M1: Having achieved an integral of the form $p(x)\ln(3x^2+5)$ (allowing for missing brackets) where $p(x)$ is constant or a polynomial in x (oe in terms of u for substitution), uses the limits within their integral - substitutes **correct limits for their variable** and subtracts, allowing either way round. The π may be missing for this mark.

A1: $\pi \ln 2$ cao. Note $\frac{\pi}{2}\ln 4$ is A0 as form is not as specified.

Question Number	Scheme	Marks
4 (a)	$a=3, b=5$	**B1**
	E.g $u=\sqrt{2x+1} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=u$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=(2x+1)^{-\frac{1}{2}}$ o.e.	**B1**
	$\int \sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x=\int 2u\ \mathrm{e}^{u}\ u\ \mathrm{d}u$	**M1**
	$=\int_3^5 2u^2\ \mathrm{e}^u\ \mathrm{d}u$	**A1**
		(4)
(b)	$\int \sqrt{8x+4}\ \mathrm{e}^{2x+1}\ \mathrm{d}x=\int 2u^2\ \mathrm{e}^u\ \mathrm{d}u$	
	$= 2u^2\mathrm{e}^u-\int 4u\ \mathrm{e}^u\ \mathrm{d}u$	**M1**
	$= =2u^2\mathrm{e}^u-\left(4u\mathrm{e}^u-4\mathrm{e}^u\right)=2u^2\mathrm{e}^u-4u\mathrm{e}^u+4\mathrm{e}^u$	**dM1 A1ft**
	$\int_4^{12}\sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x=\left[2u^2\mathrm{e}^u-4u\mathrm{e}^u+4\mathrm{e}^u\right]_3^5=\left(50\mathrm{e}^5-20\mathrm{e}^5+4\mathrm{e}^5\right)-\left(18\mathrm{e}^3-12\mathrm{e}^3+4\mathrm{e}^3\right)$	**ddM1**
	$=34\mathrm{e}^5-10\mathrm{e}^3$	**A1**
		(5)
		(9 marks)

Notes:

(a)

B1: For both $a=3, b=5$ seen in their solution. Allow if these are recovered in (b).

B1: For a correct expression involving $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$ or du and dx separately. May be unsimplified

M1: Attempts to fully change $\int \sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$ into an integral with respect to u. Must include an attempt at replacing dx to get du so M0 if there are no d terms present or dx becomes du without an attempt at connecting them first (ie there must have been an attempt at $\frac{\mathrm{d}u}{\mathrm{d}x}$ oe).

A1: Complete method to show I = $\int_3^5 2u^2\ \mathrm{e}^u\ \mathrm{d}u$. Must include the correct limits and the du.

(b) Note: you may see different ways of presenting the application of parts e.g D/I method.

M1: Use of integration by parts once to obtain $pu^2\mathrm{e}^u-\int qu\ \mathrm{e}^u\ \mathrm{d}u$, where p, $q > 0$ (if k is positive, otherwise signs will be opposite) and may be in terms of k (as can the dM mark).

dM1: Completely integrates by parts twice to a form $pu^2\mathrm{e}^u-qu\mathrm{e}^u\pm r\mathrm{e}^u$ where p, $q > 0$ (if $k > 0$ as before). Note they may evaluate in stages, but look for the complete integration overall.

A1ft: $\int_a^b ku^2\ \mathrm{e}^u\ \mathrm{d}u= ku^2\mathrm{e}^u-2ku\mathrm{e}^u+2k\mathrm{e}^u$ (oe) accepted with k or their value for k. May have the last two terms bracketed but must be seen as a complete answer in their work.

ddM1: Substitutes their a and b into a form $pu^2\mathrm{e}^u-qu\mathrm{e}^u\pm r\mathrm{e}^u$ and subtracts (either way), but must be using a value for k at this stage. May be done in stages.

A1: $34e^5 - 10e^3$ or exact equivalent in a simplified form such as $2e^3\left(17e^2 - 5\right)$

Question Number	Scheme	Marks
5 (a)	$y^2 = 2x^2 + 15x + 10y \Rightarrow 2y\frac{dy}{dx} = 4x + 15 + 10\frac{dy}{dx}$	**M1 A1**
	$(2y-10)\frac{dy}{dx} = 4x + 15 \Rightarrow \frac{dy}{dx} = \frac{4x+15}{2y-10}$ oe	**M1, A1**
		(4)
(b)	Deduces that $2y - 10 = 0 \Rightarrow y = 5$	**B1ft**
	Substitutes $y = 5$ into $y^2 = 2x^2 + 15x + 10y \Rightarrow 2x^2 + 15x + 25 = 0$ and solves for x	**M1**
	$(p =) -5, (q =) -\frac{5}{2}$	**A1**
		(3)
		(7 marks)

Notes:

(a)

M1: Correct differentiation of one of the y terms, ie $y^2 \to 2y\frac{dy}{dx}$ or $10y \to 10\frac{dy}{dx}$.

A1: Fully correct differentiation $2y\frac{dy}{dx} = 4x + 15 + 10\frac{dy}{dx}$ o.e.

M1: Rearranges to make $\frac{dy}{dx}$ the subject. The differentiated expression must contain exactly two $\frac{dy}{dx}$ terms - one from each y term, not an extra $\frac{dy}{dx} = ...$

A1: $\frac{dy}{dx} = \frac{4x+15}{2y-10}$ oe

(b)

B1ft: Deduces that $2y - 10 = 0 \Rightarrow y = 5$. Follow through on a denominator of form $ay + b$, $a, b \neq 0$. This deduction may arise from use of the symmetry of a hyperbola.

E.g. $x = 0 \Rightarrow y^2 - 10y = 0 \Rightarrow y = 0, 10$ so p, q when $y = 5$

M1: Substitutes their $y = 5$ into $y^2 = 2x^2 + 15x + 10y \left(\Rightarrow 2x^2 + 15x + 25 = 0\right)$ and solves for x (usual rules, if no working shown (by calculator) they must give correct values for their quadratic).

A1: $(p =) -5, (q =) -\frac{5}{2}$ Correct values, do not be concerned about the labels and accept if they give as the end points of the interval (ie accept if they give $\left(-5, -\frac{5}{2}\right)$ as their answer).

(b) Alt method 1

$y^2 = 2x^2 + 15x + 10y \Rightarrow y^2 - 10y - \left(2x^2 + 15x\right) = 0$

B1: Deduces that roots for x don't exist when $100 + 4\times\left(2x^2 + 15x\right) < 0$

M1: Solves their $2x^2 + 15x + 25 < 0$

A1: $\left(p =\right) - 5, \left(q =\right) - \frac{5}{2}$ Correct values, see note on main scheme.

(b) Alt method 2

B1: $y = 0 \Rightarrow 2x^2 + 15x = 0 \Rightarrow x = 0, -\frac{15}{2}$ Correct values found for x when $y = 0$

M1: Full method to use symmetry to deduce the required values of x. E.g. by symmetry, values required are one third and two thirds way between these $\Rightarrow x = \frac{1}{3}\times -\frac{15}{2}, \frac{2}{3}\times -\frac{15}{2}$

A1: $\left(p =\right) - 5, \left(q =\right) - \frac{5}{2}$ Correct values, see note on main scheme.

Question Number	Scheme	Marks
6 (a)(i)	$\overrightarrow{AB} = (\pm)\left[(8\mathbf{i}+3\mathbf{j}-7\mathbf{k})-(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})\right] = ...$	**M1**
	$\overrightarrow{AB} = 6\mathbf{i}+\mathbf{6j}-12\mathbf{k}$	**A1**
(ii)	$\mathbf{r} = \begin{pmatrix}2\\-3\\5\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-2\end{pmatrix}$ o.e. such as $\mathbf{r} = \begin{pmatrix}8\\3\\-7\end{pmatrix} + \lambda\begin{pmatrix}6\\6\\-12\end{pmatrix}$	**B1ft**
		(3)
(b)	Attempts $\pm\overrightarrow{CP} = \pm\begin{pmatrix}2+\lambda-3\\-3+\lambda-5\\5-2\lambda-2\end{pmatrix}$	**M1**
	$\overrightarrow{CP}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \Rightarrow \begin{pmatrix}\lambda-1\\\lambda-8\\-2\lambda+3\end{pmatrix}\bullet k\begin{pmatrix}1\\1\\-2\end{pmatrix} = 0 \Rightarrow 1(\lambda-1)+1(\lambda-8)-2(-2\lambda+3)=0$ Alt: $(\lambda-1)^2+(\lambda-8)^2+(-2\lambda+3)^2 = 6\lambda^2-30\lambda+74 = 6\left(\lambda-\frac{5}{2}\right)^2+\frac{73}{2}$	**dM1**
	$\Rightarrow \lambda = \frac{5}{2}$ $\left[\text{use of } \overrightarrow{AB} \text{ in } \overrightarrow{CP} \text{ gives } \lambda = \frac{5}{12}, \text{use of } \overrightarrow{OB}\ \lambda = \frac{-7}{2} \text{ or } \frac{-7}{12}\right]$	**A1**
	$\overrightarrow{OP} = \begin{pmatrix}2\\-3\\5\end{pmatrix} + \frac{5}{2}\begin{pmatrix}1\\1\\-2\end{pmatrix} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$	**ddM1, A1** **(5)**
		(8 marks)

Notes:

Accept either form of vector notation throughout. Accept with **i**,**j** and **k** in their column vectors.

(a)(i)

M1: Attempts to subtract vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way around. May be implied by two correct components.

A1: $\overrightarrow{AB} = 6\mathbf{i}+\mathbf{6j}-12\mathbf{k}$ o.e.

(a)(ii)

B1ft: Any correct equation for the line, may use a correct or follow through multiple of $\overrightarrow{AB}$ for direction and with any point on the line. Must start **r** = … or accept $x\mathbf{i}+y\mathbf{j}+z\mathbf{k} = ...$ (l = … is B0).

(b)

M1: Attempts $\pm\overrightarrow{CP}$ using point C and a general point on their l

dM1: Sets the scalar product of their $\overrightarrow{CP}$ (either direction) and their direction of l (or $\overrightarrow{AB}$) to 0 and proceeds to an equation in λ. Condone sign slips in components if the intention is clear.

Alternatively attempts to minimise the distance CP (by completing square as shown, or by differentiation) to obtain a linear equation in λ.

A1: Finds a correct value of λ for their *l*. Note if they use $\overrightarrow{AB}$ the correct value is $\frac{5}{12}$

ddM1: Substitutes their λ (from a correct method) into their *l*

A1: $\overrightarrow{OP} = \frac{9}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ Accept as coordinates, and accept $P = \ldots$ instead of $\overrightarrow{OP}$.

Question Number	Scheme	Marks
7 (a)	$\frac{dV}{dr} = 4\pi r^2$	**B1** **(1)**
(b) (i)	$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \Rightarrow V = -\frac{450}{2t+3} + c$ (oe)	**M1 A1**
	$t = 0, V = 0 \Rightarrow 0 = -\frac{450}{3} + c \Rightarrow c = ...$	**M1**
	$V = 150 - \frac{450}{2t+3} = \frac{300t + 450 - 450}{2t+3} = \frac{300t}{2t+3}$ *	**A1 ***
(ii)	150 cm^3	**B1**
		(5)
(c)	$t = 3 \Rightarrow V = \frac{300 \times 3}{2 \times 3 + 3} = (100)$	**M1**
	$100 = \frac{4}{3}\pi r^3 \Rightarrow r = 2.88$ cm	**dM1 A1cao** **(3)**
(d)	$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow \frac{900}{(2t+3)^2} = 4\pi r^2 \times \frac{dr}{dt}$	**M1**
	$t = 3, r = "2.88" \Rightarrow \frac{900}{81} = 4\pi \times 2.88^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = ...$	**dM1**
	$\Rightarrow \frac{dr}{dt} = \text{awrt}\, 0.11\ \text{cm s}^{-1}$	**A1** **(3)**
		(12 marks)

Notes: Mark the question as a whole. Penalise only once for missing/incorrect units in the question.

(a)
B1: cao See scheme.
(b)(i)
M1: Integrates to a form $V = \frac{k}{2t+3}$ (oe) with or without + *c*. Condone a sign error in $2t - 3$.
A1: $V = -\frac{450}{2t+3}(+c)$ (oe). There is no need for + *c*
M1: Substitutes $V = 0, t = 0$ and proceeds to find a value for *c*. There must have been an attempt at integrating to achieve a function in *V* and *t* with a constant of integration.
A1*: Correct integration and value for *c* with at least one intermediate step with *c* substituted back in the equation before proceeding to the given answer.
(b)(ii)
B1: 150 cm^3. Must include units.
(c)
M1: Attempts to substitute t = 3 into the equation for *V*. Allow if there is a slip in substitution.
dM1: Uses their *V* in $V = \frac{4}{3}\pi r^3$ to find a value for *r*
A1: cao $r = 2.88$ cm. Must be to 3 s.f.. Must include units unless already penalised in (b)(ii).
(d)
M1: Attempts to use $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ (oe) with the given formula for $\frac{dV}{dt}$ and an attempt at substituting their $\frac{dV}{dr}$ (allow if this substitution is not in the correct place if a correct chain rule has been stated.)

dM1: Substitutes both $t = 3$ and their value for r and proceeds to find a value for $\frac{dr}{dt}$. If no substitution shown, the answer must be correct for their r to imply the method (may need to check).
A1: awrt 0.11 cm s^{-1}. Must include units unless already penalised in (b)(ii) or (c).

Question Number	Scheme	Marks
8 (a)	At $t=\frac{\pi}{4}$ $P=\left(\frac{1}{2},2\right)$	**B1**
	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2\sec^2 t}{2\sin t\cos t}=4$ when $t=\frac{\pi}{4}$	**M1 A1**
	Equation of l: $y-2=-\frac{1}{4}\left(x-\frac{1}{2}\right)\Rightarrow 8y-16=-2x+1\Rightarrow 8y+2x=17$*	**dM1 A1 * cso**
		(5)
(b)	$\int y\frac{dx}{dt}\,dt=\int 2\tan t\times 2\sin t\cos t\,dt$	**M1**
	$=\int 4\sin^2 t\,dt$	**A1**
	$=\int 2-2\cos 2t\,dt = 2t-\sin 2t$	**dM1 A1**
	Total area of $S=\left[2t-\sin 2t\right]_0^{\frac{\pi}{4}}+\frac{1}{2}\times 8\times 2=\frac{\pi}{2}-1+8=\frac{\pi}{2}+7$	**M1 A1**
		(6)
		(11 marks)

Notes:

(a)

B1: Correct coordinates for P stated or implied by working.

M1: Attempts to find $\frac{dy}{dx}$ using $\frac{dy/dt}{dx/dt}$ at $t=\frac{\pi}{4}$. Condone poor differentiation. Substitution of the $\frac{\pi}{4}$ is sufficient for the method. Alternatively, may attempt $\frac{dx}{dy}$ or $-\frac{dx}{dy}$. Accept a value following finding $\frac{dy}{dx}$ (or its reciprocal etc) as an attempt to evaluate at $t=\frac{\pi}{4}$ if no contrary working is shown **but check carefully** as the correct answer may arise from incorrect working.

A1: Correct $\frac{dy}{dx}=4$ (oe equation) following correct differentiation. May be implied.

dM1: Attempts to find the equation of the normal at $t=\frac{\pi}{4}$. It is dependent upon the previous M and use of their P. The value of the gradient used must be correct for their differential.

A1*: **cso** Correct proof leading to $8y+2x=17$

(b)

M1: Attempts $\int y\frac{dx}{dt}\,dt=\int 2\tan t\times\text{"}2\sin t\cos t\text{"}\,dt$ with their $\frac{dx}{dt}$ condoning slips on coefficients.

A1: $\int 4\sin^2 t\,dt$

dM1: Uses $\cos 2t = \pm 1 \pm 2\sin^2 t$ and integrates $\int \pm p \pm q\cos 2t \, dt$ to a form $\pm at \pm b\sin 2t$

See note below.

A1: $\int y \frac{dx}{dt} dt = 2t - \sin 2t$ See note below.

M1: Full method to find area of region *S*. Finds the sum of their values for $\int_0^{\frac{\pi}{4}} y \frac{dx}{dt} dt$ and $\frac{1}{2}\left(\frac{17}{2} - P_x\right) \times P_y$. Condone poor integration for this mark as long as they are attempting to apply the correct limits to their result. They may attempt the area under the line by integration:

$\int_{P_x}^{\frac{17}{2}} -\frac{1}{4}x + \frac{17}{8} dx$ In such a method condone minor slips, but must be attempting correct limits.

A1: $\frac{\pi}{2} + 7$

Note: If the t's becomes x's during the integration, then allow the M's and the A's if recovered but if $2x - \sin 2x$ or with mixed variables is found and x values substituted then it is M1A0 for the integral and M0 for the method for area.

8 (a) Alt	At $t = \frac{\pi}{4}$ $P = \left(\frac{1}{2}, 2\right)$	**B1**
	$y = \frac{2\sin t}{\cos t} = \frac{2\sqrt{x}}{\sqrt{1-x}}$ $y^2 = \frac{4x}{1-x}$ $\frac{dy}{dx} = \frac{x^{-\frac{1}{2}}(1-x)^{\frac{1}{2}} - 2\sqrt{x} \times -\frac{1}{2}(1-x)^{-\frac{1}{2}}}{1-x} \Rightarrow \left.\frac{dy}{dx}\right\|_{x=\frac{1}{2}} = 4$ or $2y\frac{dy}{dx} = \frac{4(1-x) - 4x \times -1}{(1-x)^2} \Rightarrow \left.\frac{dy}{dx}\right\|_{x=\frac{1}{2}, y=2} = 4$ oe	**M1 A1**
	Equation of l: $y - 2 = -\frac{1}{4}\left(x - \frac{1}{2}\right) \Rightarrow 8y - 16 = -2x + 1 \Rightarrow 8y + 2x = 17$ *	**dM1 A1 * cso**
		(5)

(a)

B1: Correct coordinate for P $\left(\frac{1}{2}, 2\right)$ stated or implied by working.

M1: Attempts to find Cartesian equation for C, any form, and attempts $\frac{dy}{dx}$ (or an equivalent as main scheme) with appropriate differentiation methods for their Cartesian form, allowing for slips and finds x and/or y using $t = \frac{\pi}{4}$ and evaluate the derivative with these values.

A1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ (oe equation) following correct differentiation and from correct work.
dM1: Attempts to find the equation of the normal at their x and y values.
It is dependent upon the previous M and use of their P.
A1*: **cso** Correct proof leading to $8y + 2x = 17$

Question Number	Scheme	Marks
9 (a)	Let $p=3k+2$ then $(3k+2)^3 = 27k^3+54k^2+36k+8$	**M1**
	$=3\times(9k^3+18k^2+12k+3)-1$ not a multiple of 3	**A1**
	So p cannot be of form $3k+1$ or $3k+2$, since p^3 is a multiple of 3. Hence p must be a multiple of 3, a contradiction of our assumption, hence for all integers p, when p^3 is a multiple of 3, then p is a multiple of 3	**A1**
		(3)
(b)	Assumption: there exist (integers) p and q such that $\sqrt[3]{3}=\frac{p}{q}$ (where p and q have no (non-trivial) common factors.)	**B1**
	Then $\sqrt[3]{3}=\frac{p}{q} \Rightarrow p^3 = 3q^3$	**M1**
	So p^3 is a multiple of 3 and (so) p is a multiple of 3	**A1**
	But $p=3k \Rightarrow 27k^3 = 3q^3 \Rightarrow q^3 = 9k^3$	**dM1**
	Hence q^3 is a multiple of 3 so q is a multiple of 3, but as p and q have no (non-trivial) common factors, this is a contradiction. Hence $\sqrt[3]{3}$ is an irrational number.*	**A1***
		(5)
		(8 marks)

Notes:

(a)

M1: Attempts to expand $(3k+2)^3$ or $(3k-1)^3$.

Look for a cubic expression with **4 terms** with at least **two correct** (allowing for incorrect signs).

A1: Achieves a correct $3\times(...)+r, |r|<10$ form for the expansion and states not a multiple of 3.

Suitable forms include $(3k+2)^3 = 3(9k^3+18k^2+12k+2)+2 = 3(9k^3+18k^2+12k+3)-1$ or $3(9k^3+18k^2+12k)+8$ or $(3k-1)^3 = 3(9k^3-9k^2+3k)-1$ etc.

Alternatively, achieves correct $(3k+2)^3 = 27k^3+54k^2+36k+8$ or $(3k-1)^3 = 27k^3-27k^2+9k-1$ with a **reason** why it is not a multiple of 3 e.g 3 divides 27, 54 and 36, but not 8 hence not divisible by 3.

A1: Completes the proof. Must have scored both previous marks and a reference to **both cases** (in some form) leading to a "**contradiction"** and some indication that **proof is complete**. It is unlikely to be as complete as that shown in the scheme, but all three bold points must be conveyed. E.g. as a minimum after satisfying the first A "both cases give a contradiction hence the original statement is true".

(b)

May use different letters throughout.

B1: Sets up algebraically the initial statement to be contradicted. Essentially for showing they know what a rational number is algebraically. There is no requirement for this mark to state that p and q are integers with no (non-trivial) common factors (this may be implied for this mark).
M1: Cubes correctly and multiplies through by q^3
A1: Deduces that both p^3 is a multiple of 3 and hence p is a multiple of 3. Jumping directly to p is a multiple of 3 is A0.
dM1: Sets $p = 3k$ and proceeds to find q^3 in terms of k. (May use a different letter.)
A1: Completes the proof. This requires

- Correct algebraic statements
- Correct deductions in correct order. E.g. p^3 is a multiple of 3 so p is a multiple of 3
- initial statement must have included that p and q are integers (or accept natural numbers or a $\frac{p}{q}$ is a fraction) and have no common factors (or are co-prime, or in simplest form)
- correct reason for contradiction and acceptable conclusion
- There must have been no contrary statements during the proof (e.g. that p and q are prime)

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